

NASA Technical Paper 1660

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Generated by Subsonic or Supersonic
Propeller- or Rotor-Blade Tips

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National Aeronautics
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SUMMARY

The nonlinear propagation equations for sound generated by a constant-speed blade tip are presented. Propagation from a subsonic tip is treated as well as the various cases that can occur at supersonic speeds. Some computed examples indicate that the nonlinear theory correlates with experimental results better than linear theory for large-amplitude waves. For swept tips that generate a wave with large-amplitude leading expansion, the nonlinear theory predicts a cancellation effect that results in a significant reduction of both amplitude and impulse.

INTRODUCTION

When propeller- or rotor-tip speeds become transonic or supersonic, the pressure variation generated by the tip motion becomes compressed into a short time span and the amplitude becomes large. (See, for example, fig. 3 of ref. 1.) When the amplitude attains a sufficiently high level the wave begins to distort in the course of its propagation. Shock waves may form as a result of this nonlinear distortion (ref. 2).

The recent history of the development of theoretical methods for calculating tip noise includes contributions in the areas of time domain analysis with noncompactness of sources (ref. 1), frequency domain analysis with consideration of quadrupoles (ref. 3), and contributions to the nonlinear theory of propagation (ref. 4). More extensive literature surveys are given in the introductory sections of these references.

The method of reference 4 assumes an N-wave pressure signature which, according to the Whitham theory (ref. 2), is the ultimate far-field shape for compression pulses. However, for subsonic tips or even for tips at low supersonic Mach numbers, the pressure forms are generally not of the type that ever attain an N-wave form. (See, for example, the signatures in refs. 1 and 4.) Therefore, a more general signature form is treated in the present analysis. The problem of propagation from a supersonic tip is also included. This latter problem involves several different propagation possibilities depending on the orientation of the ray relative to the instantaneous plane of the motion.

The present method has thus far been applied only in conjunction with the linear source theory calculation (ref. 1). It therefore suffers from the tendency of the linear theory to underpredict the near-field peak pressure (ref. 1). However, the present propagation theory is not inherently limited to the linear source theory, but could equally well be matched with a nonlinear source theory or even near-field experimental results.

SYMBOLS

In the analysis, units are arbitrary. In the figures, pressure is measured in pascals and time in seconds.

A	ray-tube cross-sectional area
a	speed of sound in undisturbed air
b	coefficient for determining speed in z-direction (eq. (12))
F	pressure signature shape function (eq. (1))
h	$= \left \frac{Mk \cos \theta}{\beta} \right \quad (\text{eq. (8)})$
$\bar{i}, \bar{j}, \bar{k}$	orthonormal system of base vectors
k	$= \frac{\gamma + 1}{2\gamma a} \quad (\text{eq. (3)})$
M	helical Mach number
\bar{N}	unit vector normal to osculating plane
p	local pressure
p_0	pressure in undisturbed medium
R	radius from hub center to blade tip
\bar{r}	vector location of propeller or rotor tip
s	distance along ray from sound source
s_0	initial distance at which nonlinear and linear signatures are matched for propagation from subsonic tip
t	time
z	direction of linear motion of propeller or rotor
β	$= \sqrt{M^2 - 1}$
γ	ratio of specific heats
θ	angle between \bar{r} and the projection of a ray onto the plane perpendicular to the instantaneous direction of motion (see fig. 1(a))

κ	curvature of tip trajectory
σ	dummy integration variable
τ	time parameter (defined by eq. (2))
φ	angle between osculating plane of tip motion and $z = 0$ plane
ω	angular velocity

ANALYSIS

The following analysis assumes a blade for which the tip is moving at constant speed relative to the ambient air. Thus, it is applicable to a propeller on an airplane moving at constant forward speed or to a helicopter that is hovering or in vertical motion. The latter case is essentially academic, but the analysis lays a foundation for the practical case of a helicopter in forward motion.

According to reference 2, the equation for the relative overpressure in a propagating wave is

$$\frac{\Delta p}{p_0} = \frac{F(\tau)}{\sqrt{A}} \quad (1)$$

where A is the ray-tube area, F is a shape function, and the variable τ is related to the retarded time by the nonlinear equation

$$\tau = t - \frac{s}{a} + \frac{kF(\tau)}{\sqrt{A(s)}} \int_0^s \frac{d\sigma}{\sqrt{A(\sigma)}} \quad (2)$$

In this equation,

$$k = \frac{\gamma + 1}{2\gamma a} \quad (3)$$

For the analysis of sonic booms propagating from supersonic airplanes, the shape function $F(\tau)$ is determined directly from the vehicle axial area and lift distributions (ref. 5). However, for the analysis of tip noise, one can take advantage of the existing linear theory and work directly from the linear calculation of the near-field signature (refs. 1 and 4). Thus, if the signature calculated by linear theory is

$$\frac{\Delta p}{p_0} \left(t - \frac{s}{a} \right) \quad (4)$$

then the F-function is determined by

$$F \left(t - \frac{s}{a} \right) = \sqrt{A(s)} \frac{\Delta p}{p_0} \left(t - \frac{s}{a} \right) \quad (5)$$

according to equation (1). Then, at any distance s from the sound source, the nonlinear wave can be calculated by means of equations (1) and (2), as follows. At a given distance s , these two equations represent a linear transformation

from the old system of coordinates $\left(t - \frac{s}{a}, F \right)$ to the new system $\left(\tau, \frac{\Delta p}{p_0} \right)$. When

the known curve $F \left(t - \frac{s}{a} \right)$ is transformed by these equations, the ordinates of

each point are multiplied by the factor $1/\sqrt{A}$. (See eq. (1).) The abscissas are shifted to the left by an amount proportional to the ordinate F , according to the last term in equation (2). If the value of F is sufficiently large (corresponding to a high overpressure), then after a certain distance this shifting of abscissas causes the resulting curve to represent a multivalued function of τ . Such a function cannot correspond physically to an acoustic wave shape. However, if discontinuities corresponding to shock waves are inserted in the multivalued regions according to the equal-area rule (ref. 2), then the resulting function corresponds to the actual wave shape.

Before this calculation can be made, the ray-tube area $A(s)$ and the integral $\int_0^s \frac{d\sigma}{\sqrt{A(\sigma)}}$ must be determined. If the tip Mach number is subsonic, the propagation is always spherical. Thus, the ray-tube area varies as s^2 . However, as has been observed in reference 2, for this case the lower limit of the integral in equation (2) cannot be taken to be zero. Equation (2) is replaced with

$$t = \frac{s - s_0}{a} - kF(\tau) \log \left(\frac{s}{s_0} \right) + \tau \quad (6)$$

where the pressure signature is matched with its linear expression at some small initial distance s_0 . This device may at first appear somewhat arbitrary, but it appears that the far-field signature is relatively independent of the value of s_0 provided that it is sufficiently small.

At supersonic tip speeds the calculation of the nonlinear distortion term becomes more involved than at subsonic speeds. The ray-tube area is dependent both on the trajectory of the source and on the orientation of the ray tube relative to the plane of the trajectory. (See fig. 1.) According to reference 6

$$A \approx s \left(1 + \frac{M\kappa \cos \theta}{\beta} s \right) \quad (7)$$

where θ is the angle shown in figure 1(a) and κ is the curvature of the source path. In this equation, a proportionality constant containing the factor β^2/M^2 has been omitted. Since only the variation of the ray-tube area is used in the calculation, this omission is unimportant except in the case $M \rightarrow 1.0$. Then $\beta \rightarrow 0$ and the variation of the ray-tube area vanishes. That is, the rays, which are the orthogonal trajectories of the wave envelope, are parallel. This result, of course, is consistent with the fact that the wave envelope for $M = 1.0$ is a plane wave.

Define

$$h = \left| \frac{M\kappa \cos \theta}{\beta} \right| \quad (8)$$

Then equation (7) becomes

$$A = s + hs^2 \quad \left(|\theta| < \frac{\pi}{2} \right) \quad (9a)$$

$$A = s \quad \left(|\theta| = \frac{\pi}{2} \right) \quad (9b)$$

$$A = s - hs^2 \quad \left(|\theta| > \frac{\pi}{2} \right) \quad (9c)$$

These formulas provide interesting insights into the nature of the propagation from a supersonic tip. First, bearing in mind that linear variation of A with s is characteristic of cylindrical spreading and that quadratic variation is typical of spherical spreading, one observes that equation (9a) represents a combination of both types. The situation is very different from that of a

supersonic airplane performing a turning maneuver. For the airplane, κ is always quite small, M is more likely to be well above 1.0, and the rays that are associated with large ground overpressures are normally those for which $|\cos \theta|$ is well below 1.0. All of these factors tend to cause h to be small. Therefore, the last term in equation (9a) would be small in the near field (for small values of s) and the propagation would be essentially cylindrical there.

In contrast, for a propeller with a supersonic tip, κ is large, M/β is large for all cases of interest, and rays corresponding to values of $|\cos \theta|$ near unity (those near the plane of motion) strike the ground. Consequently, for the propeller tip the spherical-spreading term in equation (9a) is important even in the near field, and it predominates at a distance of a few blade diameters.

For a helicopter either hovering or in vertical motion, the ground noise is determined largely by those rays at a large angle to the plane of the tip motion (small values of $|\cos \theta|$), for which the spreading is more nearly cylindrical. In fact, for $|\cos \theta| = 0$ (directly beneath the tip of a hovering helicopter, for example), the spreading is exactly cylindrical, according to equation (9b).

Equation (9c) indicates that since A vanishes at $s = 1/h$, the rays that propagate from the inside of the turning tip always focus. For a propeller, for large values of h and small values of θ , the focal point is near the source. For a helicopter, on the other hand, the focal point would normally exist at ground level. Beyond the focus the spreading eventually becomes spherical.

It now remains to evaluate the integral in the nonlinear term in equation (2). For $\cos \theta > 0$, corresponding to equation (9a)

$$\int_0^s \frac{d\sigma}{\sqrt{A(\sigma)}} = \frac{2}{\sqrt{h}} \ln (hs + \sqrt{1 + hs}) \quad (10)$$

Thus, equation (2) becomes

$$\tau = t - \frac{s}{a} + \frac{2kF(\tau)}{\sqrt{h}} \ln (hs + \sqrt{1 + hs}) \quad (\cos \theta > 0) \quad (11a)$$

Similarly,

$$\tau = t - \frac{s}{a} + 2kF(\tau)\sqrt{s} \quad (\cos \theta = 0) \quad (11b)$$

$$\tau = t - \frac{s}{a} + \frac{2kF(\tau)}{\sqrt{h}} \tan^{-1} \sqrt{\frac{hs}{1 - hs}} \quad (\cos \theta < 0) \quad (11c)$$

A further observation should be made regarding the quantity $\cos \theta$, which governs the type of propagation in accordance with equations (9). For a propeller in forward motion the tip follows a helical path. The plane of motion of the tip is therefore not the instantaneous plane of the blades, but the osculating plane of the helix traced by the tip (ref. 7). If the plane of the blades is taken to be parallel to the $z = 0$ plane, then the equation of the helix (ref. 7) is

$$\bar{r} = R \cos \omega t \bar{i} + R \sin \omega t \bar{j} + b\omega k \quad (12)$$

The vector $\dot{\bar{r}} \times \ddot{\bar{r}}$ is perpendicular to the osculating plane (ref. 7, p. 10). (Superscript dots indicate differentiation with respect to time.) Computing the derivatives from equation (12) yields for the unit vector normal to the osculating plane

$$\bar{N} = \frac{\dot{\bar{r}} \times \ddot{\bar{r}}}{|\dot{\bar{r}} \times \ddot{\bar{r}}|} = \frac{b \sin \omega t \bar{i} - b \cos \omega t \bar{j} + R\bar{k}}{\sqrt{b^2 + R^2}} \quad (13)$$

Therefore the angle φ that the osculating plane makes with the plane of the blades is determined by

$$\cos \varphi = \bar{N} \cdot \bar{k} = \frac{R}{\sqrt{b^2 + R^2}} \quad (14)$$

DISCUSSION AND EXAMPLES

The nonlinear distortion of the wave is accounted for by the last term in equation (2), and consequently the various quantities in this term are those that determine the magnitude of the nonlinear effects. Generally, nonlinear effects become important for relative overpressures ($\Delta p/p_0$) above about 1.0×10^{-3} . According to equation (1), the amplitude is determined both by the F-function and the ray-tube area. The latter depends on the spreading law - whether it is spherical (subsonic tip), cylindrical (eq. (9b)), or a combination (eqs. (9a) and (9c)). Other factors influencing the magnitude of the nonlinear effects are the propagation distance s and the initial shape of the wave.

One of the most important of the nonlinear effects is the formation of shock waves in the signature. As the wavelets in the vicinity of the shock

pass into the shock, the wave tends to lose its original character in this region. If the F-function has regions of steep slope, the nonlinear distortion results in shock formation after a short propagation distance. These shocks, which are apparent in the nonlinear calculation for the example shown in figure 2, account for the major part of the discrepancy between the experiment and the linear-theory calculation taken from reference 8. The test signature and the nonlinear theory both indicate a lower amplitude and a somewhat longer wave than is predicted by the linear theory. This example uses the unswept SR-2 blade. (For further design details as well as those for the SR-1 and SR-3 blades, see ref. 9.)

If the slopes of the F-function curve are more gradual, the shocks are formed at a greater distance whereas the near-field signature retains most of the character of the original wave. This type of wave appears to be characteristic of sound generated by subsonic tips or by supersonic tips with the leading edges swept well behind the Mach line. In figure 3 (SR-1 blade, $M = 1.05$), the linear-theory calculation is taken from reference 3. Figure 3(a) indicates that the linear theory correlates well with the nonlinear theory in the very near field. Reference 3 indicates that, for this example, the linear theory also correlates well with the test result in the near field. However, as the wave propagates farther and shocks form, the linear theory departs radically from the nonlinear results, as shown in figure 3(b). Thus, the linear theory cannot be expected to predict the far-field noise of large-amplitude waves even though it correlates well with some near-field test results.

The wave forms generated by blade tips with the leading edge swept behind the Mach line display an interesting characteristic that leads to a significant reduction of the noise as compared with unswept blades. The effect of the sweep is to lessen the initial compression and in some cases almost to eliminate it. The large-amplitude part of the wave occurs as a leading expansion followed by a large compression. (See linear theory calculation in figs. 3 and 4.) In such a wave, the nonlinear distortion causes the large-amplitude expansion to lag in the propagation process, whereas the subsequent compression tends to advance. Thus, the compression tends to override the expansion, and a partial cancellation of the wave occurs in accordance with the equal-area rule (ref. 2). The result is a significant reduction both in amplitude and impulse (see example of fig. 4). This cancellation effect is not predicted by the linear theory, which accounts only for the reduction in amplitude due to spreading. It may also be observed that the absence of a leading shock in the examples of figures 3 and 4 precludes a rapid lengthening of the wave, as in the example of figure 2.

CONCLUDING REMARKS

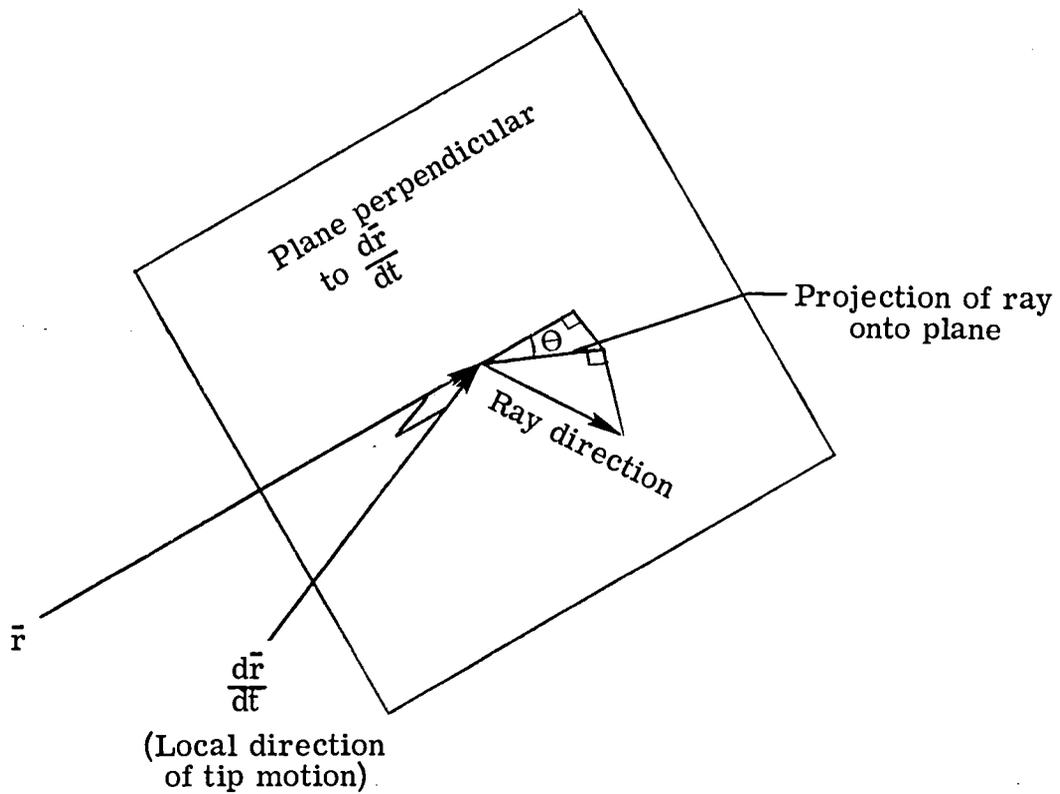
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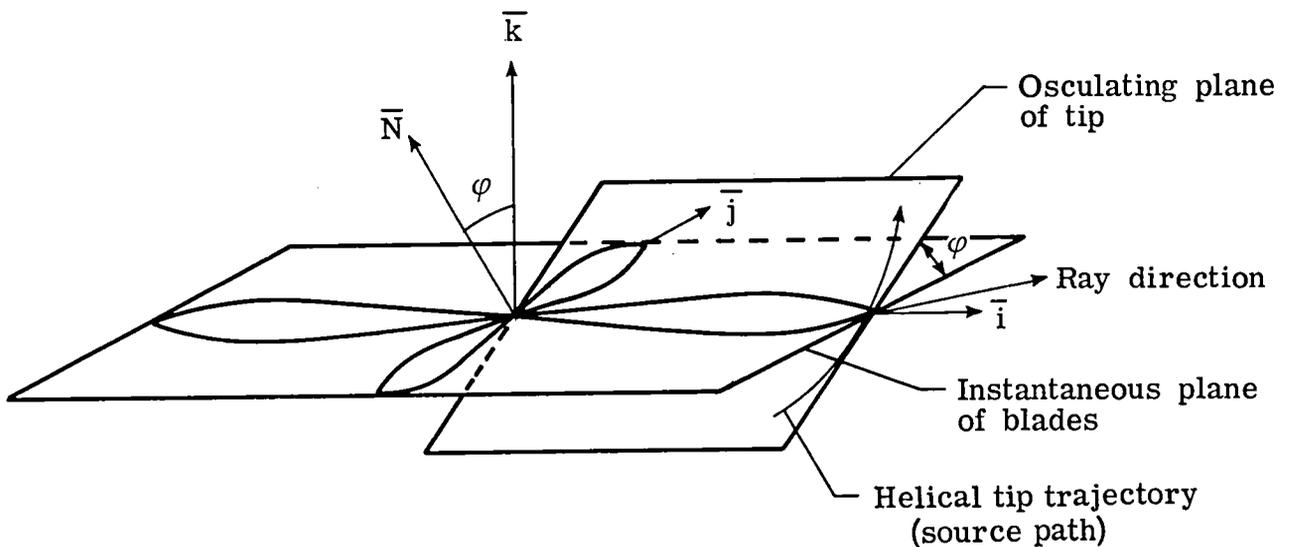
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(a) Relation of angle θ to tip vector and ray direction.



(b) Relation between instantaneous plane of blades and osculating plane of tip motion.

Figure 1.- Tip and ray geometry.

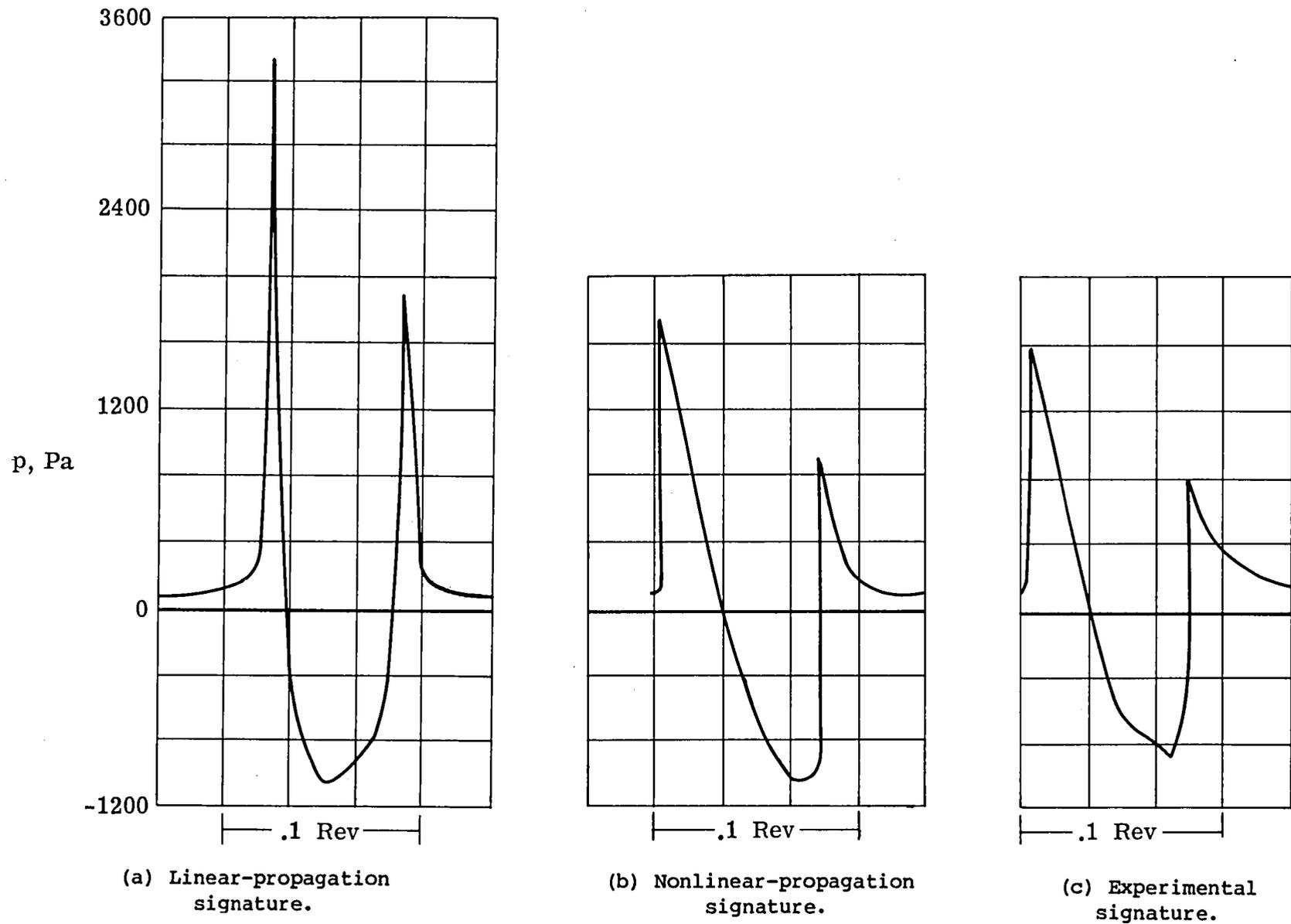


Figure 2.- Theoretical and experimental signatures and spectra for straight SR-2 blade (linear theory and experiment from ref. 8). $M = 1.18$; $s = 0.8$ diam.

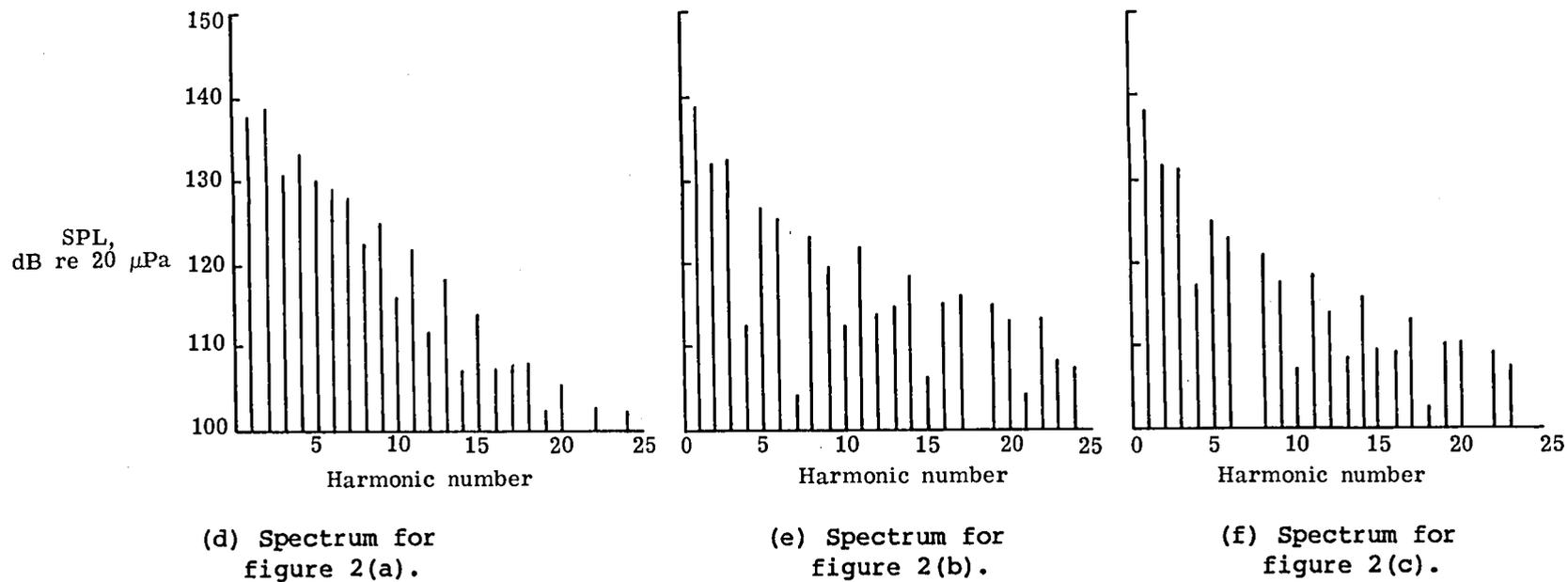
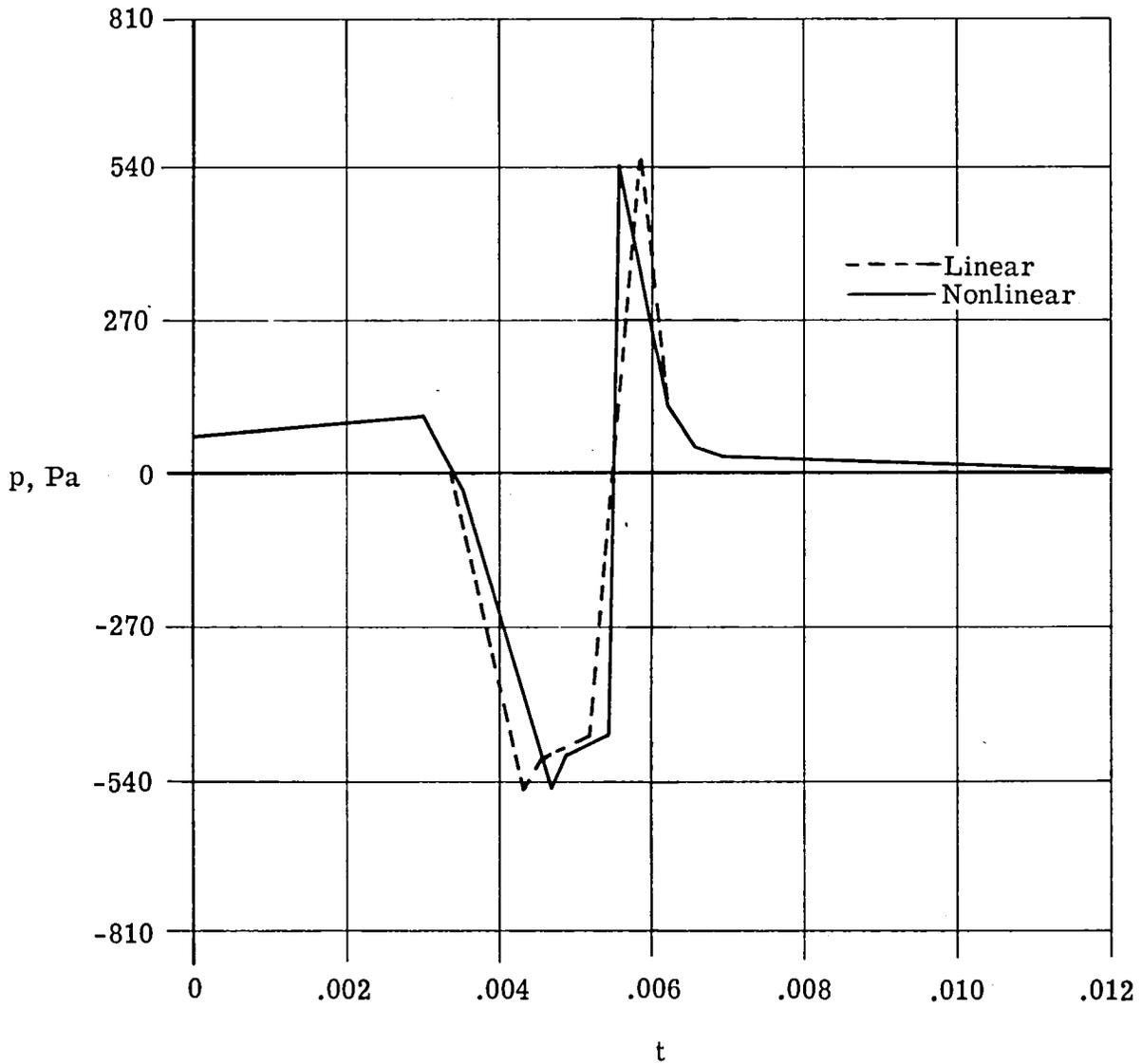
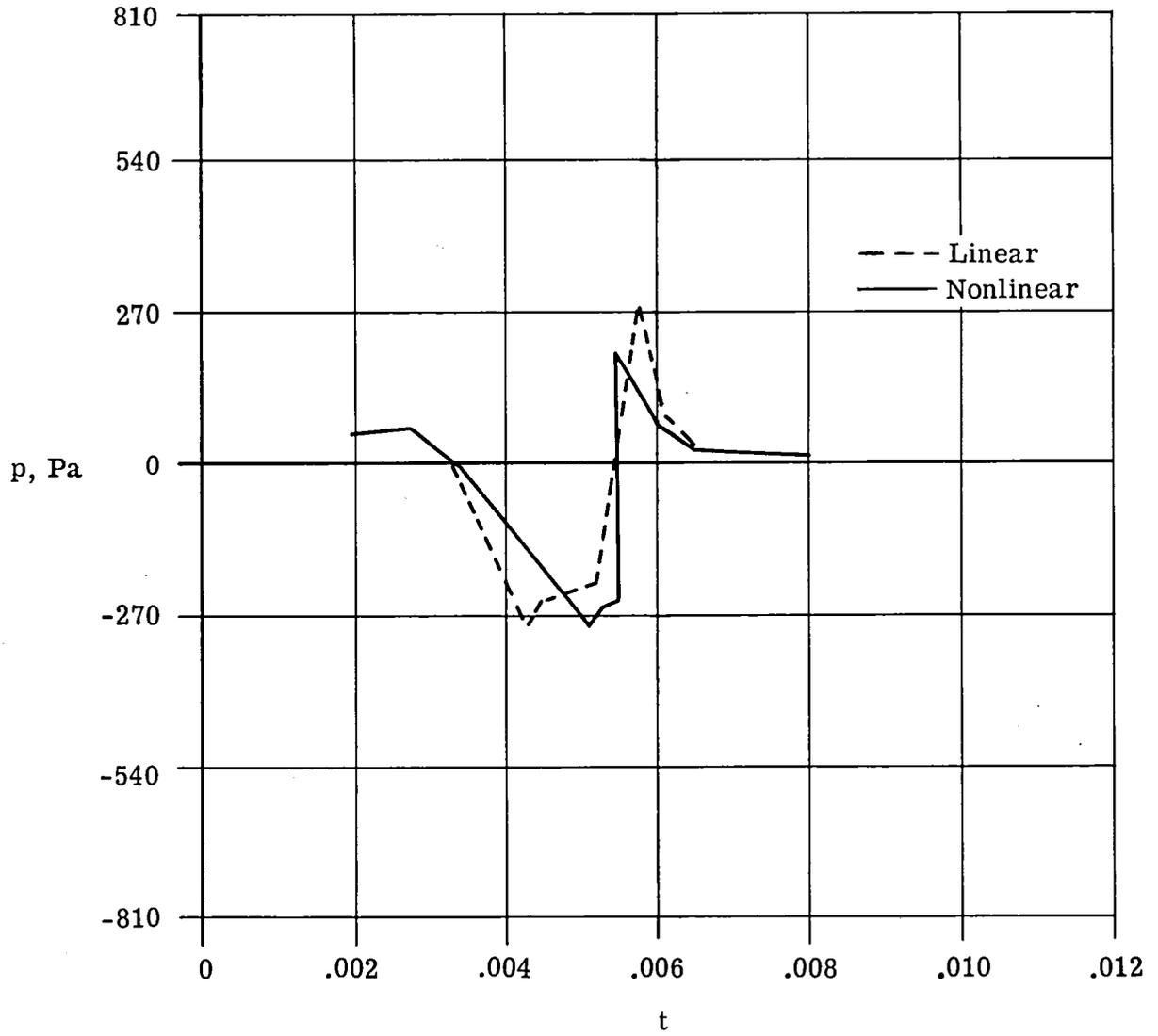


Figure 2.- Concluded.



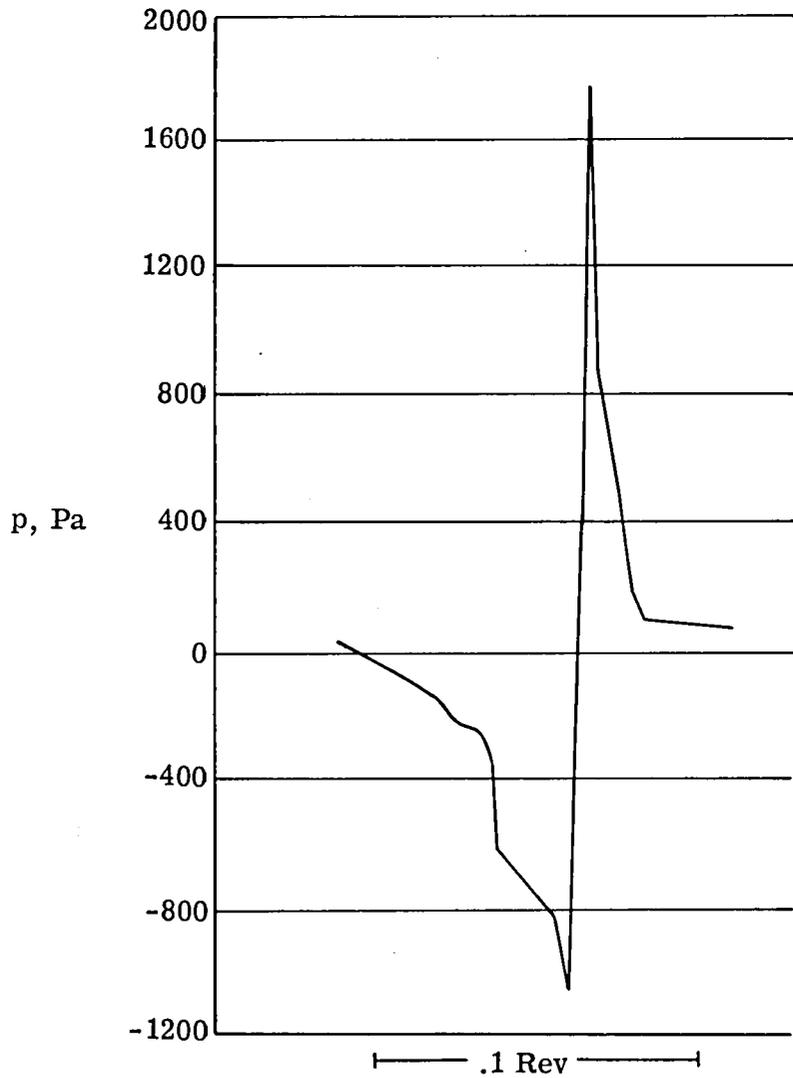
(a) $s = 0.8$ diam.

Figure 3.- Comparison of nonlinear and linear propagation calculations for SR-1 prop fan (linear results from ref. 3). Blade tips swept 30° ; $M = 1.05$.

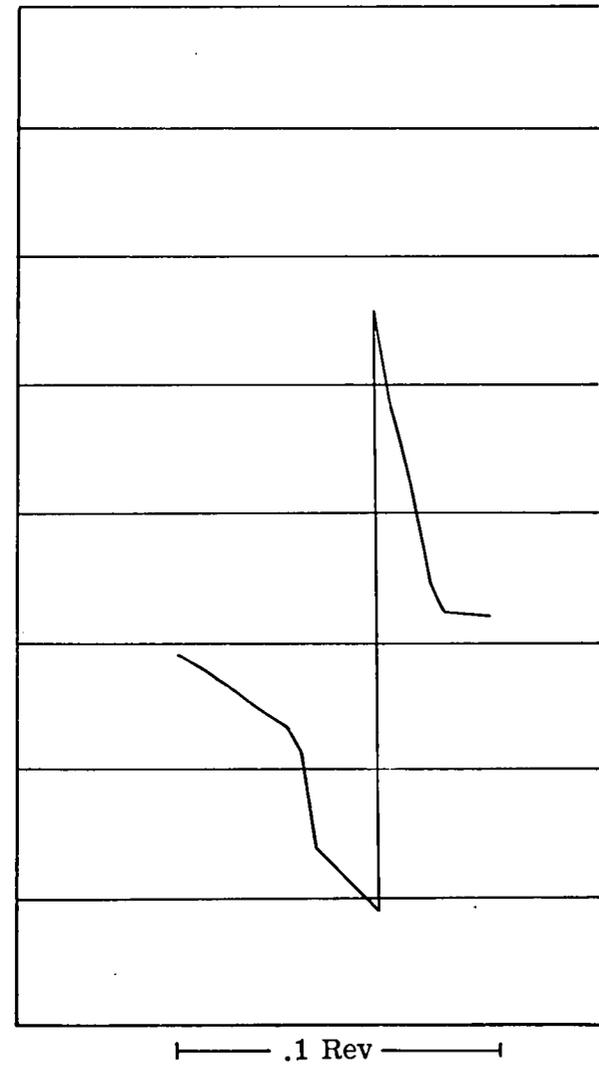


(b) $s = 1.6$ diam.

Figure 3.- Concluded.



(a) Linear theory.



(b) Nonlinear theory.

Figure 4.- Comparison of linear and nonlinear propagation theories for SR-3 swept blade tip.
 $M = 1.17$; $s = 0.8$ diam.

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